Estimation on a jump diffusion process with jumps driven by a Hawkes process

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Abstract:

In neurosciences, two kinds of data are available: a continuous signal coming from the action potential of one single neuron along time, and several discrete signals which are the spike train of several neurons (the occurring times when the action potential overcomes a threshold). Usually these two kinds of data are investigated separately using either diffusion processes to describe the dynamic of the action potential or counting processes to describe the graph of connectivity between neurons (see [3]).

We propose a new model which allows to deal with both type of signals in a single procedure to take into accounts the whole information. This new approach can be applied to neurosciences issues but also to other fields as in finance to model financial assets. Our model is a jump-diffusion model driven by a multidimensional Hawkes process (see [1]). More precisely he dynamic along time of a process $X$ is described by the following equation

$$dX_t = b(X_t)dt + \sigma(X_t)dW_t + \sum_{j=1}^{M} a(X_{t^-})dN^j_t, \quad \forall t \geq 0$$

with $(N^j)_{j=1,...,M}$ a multivariate Hawkes process with conditional intensity functions $(\lambda^j)_{j=1,...,M}$.

Assuming that we have high frequencies observations of the process together with observations of the multidimensional Hawkes process, we focus first on the nonparametric estimation of the drift function $b$, when the other coefficients are assumed to be known (see [2]) and then we consider a simultaneous estimation of both parameters $a$ and $b$, assuming that $\sigma$ is known. The proposed method is based on a mean squared regression approach. The novelty here lies in the contrast function which takes into account the multidimensional Hawkes process. We will also discuss about the existence of a stationary measure for our process and ergodicity results that we need to obtain oracle inequalities for our estimators.

Finally, a numerical study illustrates our purpose.

References